RESEARCHERID
THOMSON REUTERS

[Vyas * *et al.*, 7(3): March, 2018]
IC™ Value: 3.00

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

IJESRT
INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

FIXED POINT THEOREMS FOR PROBABILISTIC DENSIFYING MAPPINGS

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DOI: 10.5281/zenodo.1207007

ABSTRACT

Main objective of this paper is to demonstrate some quirk and common fixed point theorems for probabilistic densifying mappings. The concept of Kuratowski function on a probabilistic metric space as a generalisation of Kuratowski measure of non-compactness were introduced by Bocsan and Constantin[1]. The major role of densifying mappings in the study of fixed point theory in metric and normed linear spaces is well –known. However, the concept of probabilistic densifying mapping was introduced by Bocsan[2] and sometimes Khan and Fisher[3] proved some fixed point theorems for densifying mappings and the results so obtained were generalized by Hadzic[4].Iseki[5] gave a result which is generalisation of the result of Furi and Vignoli[6] in densifying mappings and further Iseki's results are generalized by Jain and Dixit[7] in same mappings.

Keywords: fixed point, densifying, mapping.

I. INTRODUCTION

The concept of an abstract metric space was first introduced by Frechet[8]introduced by and is a natural setting for a large number of mathematical, physical and scientific problems in which the notion of distance appear. According to him " For any two points in the space, there is a single non negative real number with certain conditions called the distance between the points" is an essential feature of this concept.Contraction mapping principlegiven by Banach is one of the most stimulating tools in applied mathematics. In this era so many generalizations of Banach contraction mapping principle have appeared.

II. PRELIMINARIES

Kuratowski [8] introduced the notion of measure of non-compactness of a bounded subset of a metric space. Further, this study was carried on by Furi and Vignoli [9]. They introduced the notion of densifying (also called condensing) mapping in terms of Kuratowski's measure of non-compactness and obtained some fixed point theorems. Following Furi and Vignoli [9], a number of mathematicians worked on densifying mappings and proved some metrical fixed point theorem. As a generalization of Kuratowski's measure of non-compactness, Bocsan and Constantin[10] introduced the notion of Kuratowski's measure of no compactness in PM-spaces. Subsequently, Bocşan studied the notion of probabilistic densifying mappings.Later, Hadžić, Tan, Chamola et al.Dimri and Pant, Pant et al. Pant et al. and Singh and Pant proved some results for such mappings. In Gangulyet.al. introduced the notion of probabilistic nearly densifying mappings and proved some interesting results in this setting.

III. APPLICATIONS

The first result on fixed point theory was given by Sehgal and Bharucha-Reid [3] wherein the notion of probabilistic contraction was introduced as a generalization of the classical Banach fixed point principle in terms of probabilistic settings. Here are some definitions related to the title of present paper.

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Definition 1.
Let X be anon empty set and F be amapping from XxX to Z, the collection of all distribution functions. The ordered pair (X+ ʃ,)is called aprobabilistic metric spsce if it satisfies the following conditions in which $F_{p\,q}$ denotes this distribution function F(p,q).

$$P_1 \;:\; F_{pq}\,(x) = 1 for\ all\ x > 0\ \ iff\ \ p = q\,;$$

$$P_2 \;:\; F_{pq}\,(0) = 0\ for\ all\ \ p, q\ \epsilon\ X$$

$$P_3 \;:\; F_{pq}\,(x) = F_{pq}\,(x)\ for\ all\ \ p, q\ \epsilon\ X$$

$$P_4 \;:\; If\ \ F_{pq}\,(x) = 1\ and\ F_{q,r}\ \ (y) = 1 then\ F_{q,r}\ \ (x + y) = 1.$$

A semigroup G is said to be left reversible if for any

r, s∈G there exist a, b∈G such that ra = sb.

Definition 2.
Let X be any non empty set .A triple (X ʃ, t) where (X ʃ) be aPM-space and 't norm' is called a Menger space if it satisfies the following condition.

$$M_{\ 1} \;:\; F_{pq}\,(x) = 1 for\ all\ x > 0\ \ iff\ \ p = q\,;$$

$$M_{\ 2} \;:\; F_{pq}\,(0) = 0\ for\ all\ \ p, q\ \epsilon\ X$$

$$M_{\ 3} \;:\; F_{pq}\ \ = F_{pq}\,(x)\ for\ all\ \ p, q\ \epsilon\ X$$

$$M_{\ 4} \;:\; F_{pq}\,(x + y) \geq t\left\{F_{q,q}\ \ (x), = 1 then\ F_{q,r}\ \ (y)\right\};$$

For all p,q,r ϵ X and x, y ≥ 0.

Remark –Here M_4 is called Menger's triangle inequality.

Definition 3.
Let A be a non empty subset of X. A function D_A (x) defined by

D_A(x) = Sup inf $F_{p\,q}$(t)
t< x p, q ϵ A

Is called the probabilistic diameter of A.
A is said to be bounded if Sup DA (x) = 1,R being the set of real numbers
x ϵ R

Definition 4.
Let(X ʃ) be a probabilistic metric space.A continuous mapping ϕ of X into X is called a probabilistic densifying mapping iff for every subset A of X such that.
α_{A}< H we have α_{ϕ} (A) < α_A

IV. RESULT AND DISCUSSION
Thus these are the supportive evidences of the title of the present paper which helps to introduce the notion of densifying (also called condensing) mapping in terms of Kuratowski'smeasure of non-compactness and obtained some fixed point theorems.

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CITE AN ARTICLE

Vyas, R. (n.d.). FIXED POINT THEOREMS FOR PROBABILISTIC DENSIFYING MAPPINGS. *INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY*, *7*(3), 618-620.